

$$\begin{aligned}
 AB^2 &= (a-c)^2 + (b-d)^2 \\
 &= (a^2 - 2ac + c^2) + (b^2 - 2bd + d^2) \\
 \because OA^2 &= OB^2 = a^2 + b^2 = c^2 + d^2 = 1 \text{ であるから} \\
 AB^2 &= a^2 + b^2 + c^2 + d^2 - 2ac - 2bd \\
 &= 2 - 2(ac + bd) \dots \textcircled{1}
 \end{aligned}$$

一方、余弦定理より、 $\triangle OAB$ において

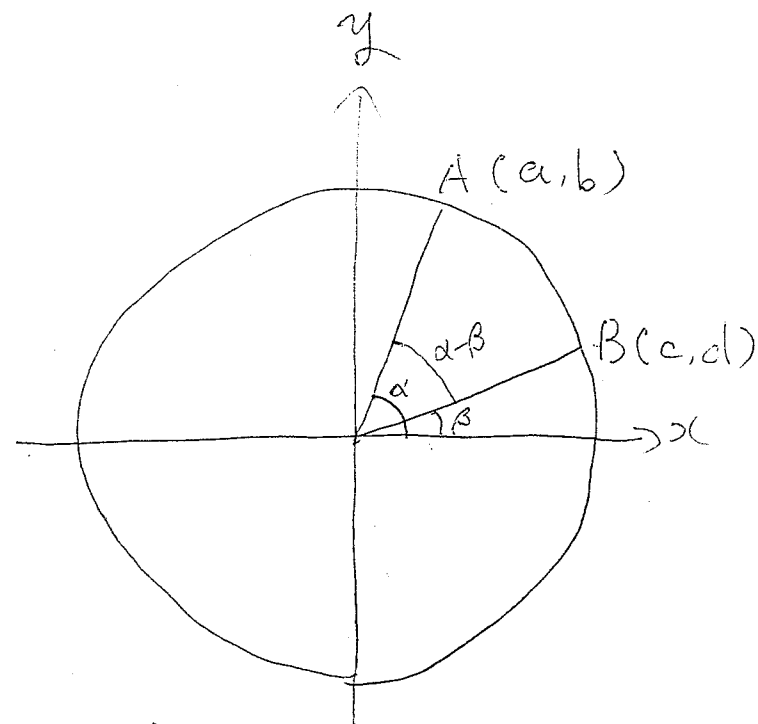
$$\begin{aligned}
 AB^2 &= OA^2 + OB^2 - 2 \cdot OA \cdot OB \cdot \cos(\alpha - \beta) \\
 &= 2 - 2 \cdot \cos(\alpha - \beta) \dots \textcircled{2}
 \end{aligned}$$

①, ②より、 $AB^2 = 2 - 2(ac + bd) = 2 - 2 \cdot \cos(\alpha - \beta)$

ゆえに $\cos(\alpha - \beta) = ac + bd$ である。

$\because a = \cos\alpha, b = \sin\alpha, c = \cos\beta, d = \sin\beta$ であるから

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta \text{ である}$$



$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

をもちに, $\cos(\alpha - \beta)$ 以外の加法定理を考へる。

前提として

$$\begin{cases} \sin(-\theta) = -\sin\theta, \cos(-\theta) = \cos\theta \dots \textcircled{1} \\ \sin(90-\theta) = \cos\theta, \cos(90-\theta) = \sin\theta \dots \textcircled{2} \end{cases}$$

$$\begin{cases} \sin(90-\theta) = \cos\theta, \cos(90-\theta) = \sin\theta \dots \textcircled{2} \end{cases}$$

を定理として扱う。

$$\circ \cos(\alpha + \beta) = \cos\{\alpha - (-\beta)\}$$

$$= \cos\alpha \cdot \cos(-\beta) + \sin\alpha \cdot \sin(-\beta)$$

$$\textcircled{1} \text{より}, \cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta \dots \textcircled{A}$$

$$\circ \sin(\alpha + \beta) = \cos\{90 - (\alpha + \beta)\} \quad \therefore \textcircled{2}$$

$$= \cos\{(90 - \alpha) - \beta\}$$

$$= \cos(90 - \alpha) \cdot \cos\beta + \sin(90 - \alpha) \cdot \sin\beta$$

$$\textcircled{2} \text{より}, \sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta \dots \textcircled{B}$$

$$\circ \sin(\alpha - \beta) = \cos\{90 - (\alpha - \beta)\} \quad \therefore \textcircled{2}$$

$$= \cos\{(90 - \alpha) + \beta\}$$

$$\textcircled{A} \text{より}, \cos\{(90 - \alpha) + \beta\} = \cos(90 - \alpha) \cdot \cos\beta - \sin(90 - \alpha) \cdot \sin\beta \\ = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta \quad \therefore \textcircled{2}$$

又して

$$\sin(\alpha - \beta) = \sin\{\alpha + (-\beta)\}$$

$$\textcircled{B} \text{より}, \sin\{\alpha + (-\beta)\} = \sin\alpha \cdot \cos(-\beta) + \cos\alpha \cdot \sin(-\beta)$$

$$= \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta \quad \therefore \textcircled{1}$$

さらに、113113+2: 定理を考えてみる

$$\begin{aligned}\circ \tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\ &= \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}\end{aligned}$$

二二二: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ という公式を利用してさらに、分母と分子をそれぞれ $\cos\alpha \cdot \cos\beta$ で割ると、

$$\begin{aligned}\frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta} &= \frac{\frac{\sin\alpha \cdot \cos\beta}{\cos\alpha \cdot \cos\beta} + \frac{\cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta}}{\frac{\cos\alpha \cdot \cos\beta}{\cos\alpha \cdot \cos\beta} - \frac{\sin\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta}} \\ &= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\sin\beta}{\cos\beta}} \\ &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}\end{aligned}$$